Vertical Stress Increases in Soil

Types of Loading

Point Loads (P)

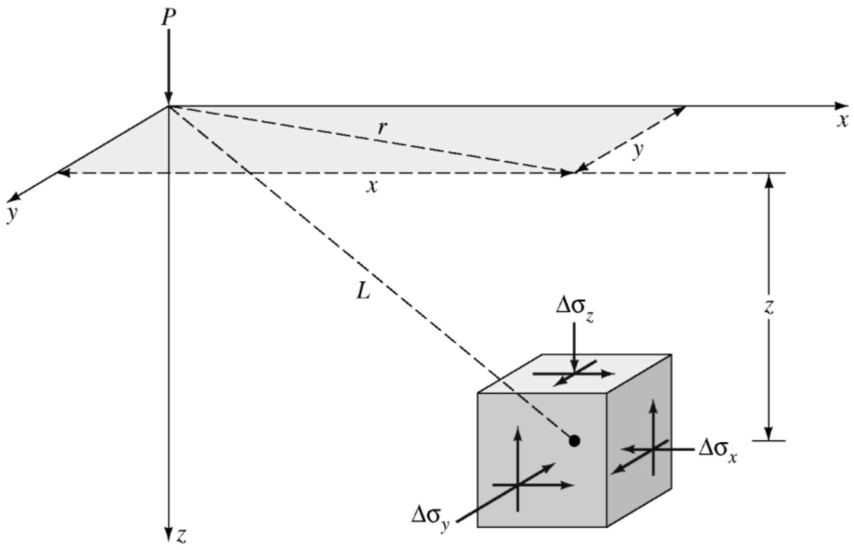


Figure 6.11. Das FGE (2005).

Examples:

- Posts

Line Loads (q/unit length)

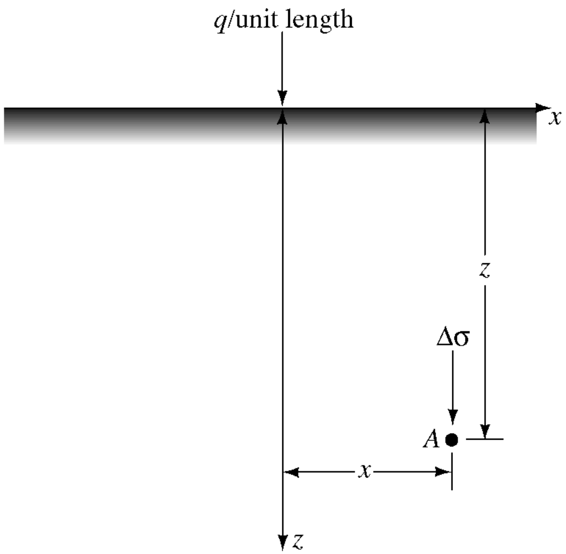


Figure 6.12. Das FGE (2005).

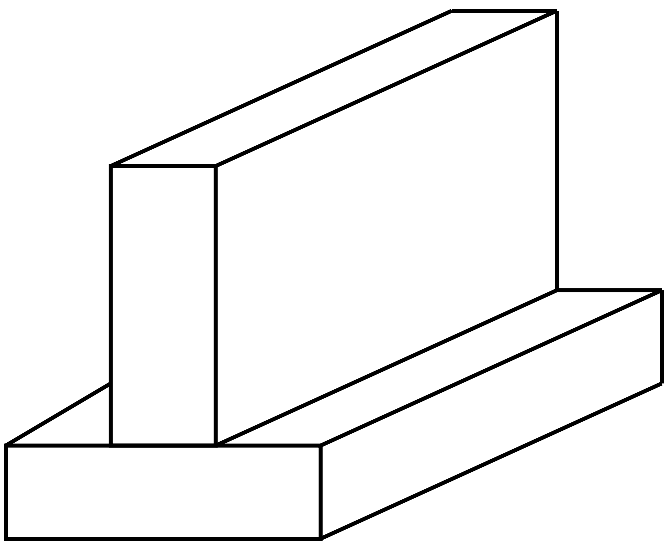
Examples:

- Railroad track

VERTICAL STRESS INCREASES IN SOIL

TYPES OF LOADING

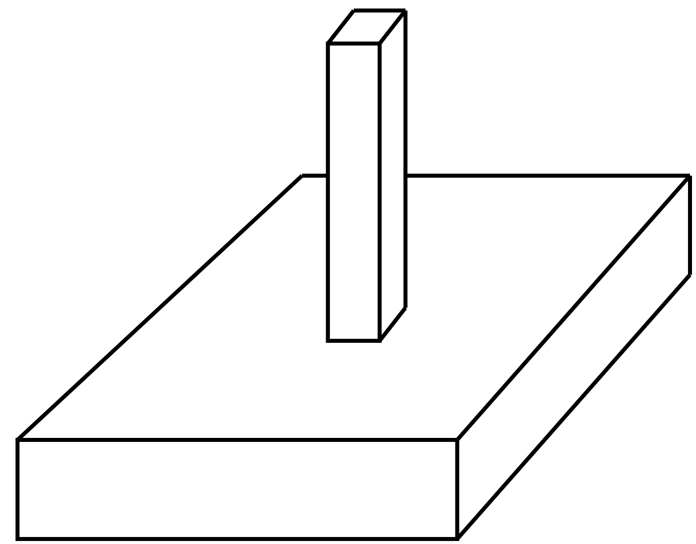
Strip Loads (q)



Examples:

- Exterior Wall Foundations

Area Loads (q)



Examples:

- Column Footings

VERTICAL STRESS INCREASES IN SOIL

ANALYSIS METHODS: BOUSSINESQ (1993)

Based on homogeneous, weightless, elastic, isotropic infinitely large half-space free of initial stress and deformation. The modulus of elasticity is assumed constant and the principle of linear superposition is assumed valid (EM1110-1-1904, 1990). Not accurate for layered soil stratigraphy with substantial thickness (NAVFAC DM7.01, 1986).

Rigid Surface Layer Over Weaker Underlying Layer: If the surface layer is the more rigid, it acts as a distributing mat and the vertical stresses in the underlying soil layer are ***less than Boussinesq values*.**

Weaker Surface Layer Over Stronger Underlying Layers: If the surface layer is less rigid than the underlying layer, then vertical stresses in both layers ***exceed the Boussinesq values*.**

Vertical Stress Increases in Soil

Analysis Methods: Westergaard

Based on the assumption that the soil on which load is applied is reinforced by closely spaced horizontal layers which prevent horizontal displacement. The effect of the Westergaard assumption is to ***reduce the stresses substantially below those obtained by the Boussinesq equations***.

Vertical Stress Increases in Soil

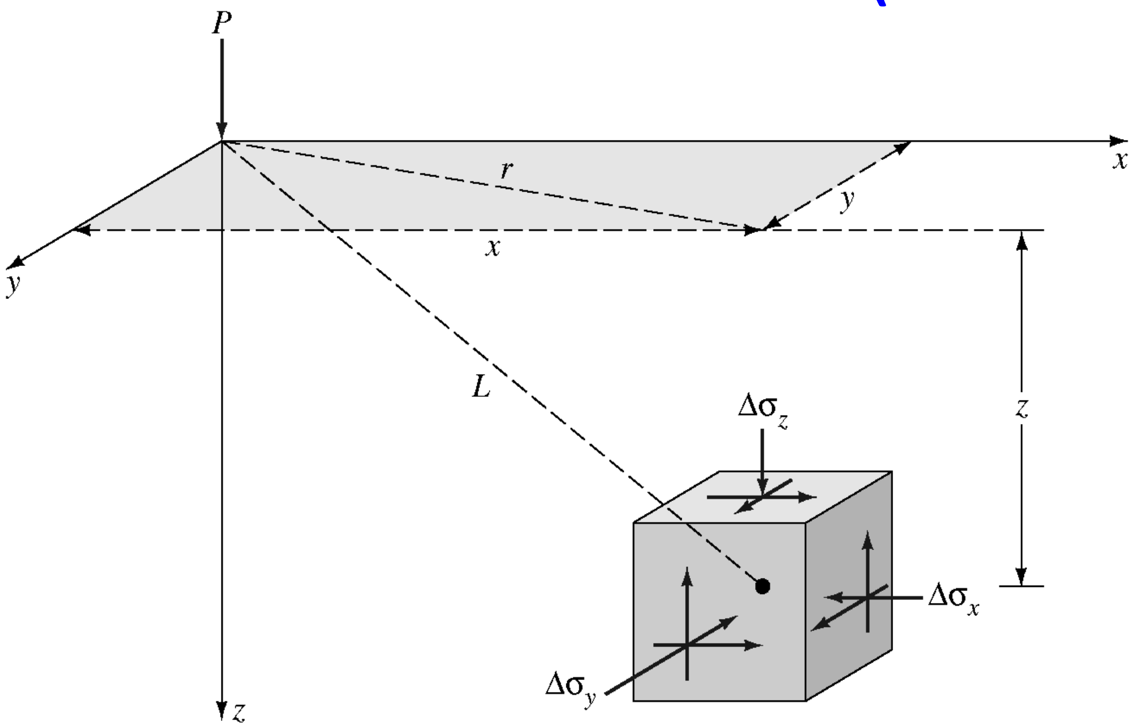
Analysis Methods: 2V:1H Method

An approximate stress distribution assumes that the total applied load on the surface of the soil is distributed over an area of the same shape as the loaded area on the surface, but with dimensions that increase by an amount equal to the depth below the surface.

Vertical stresses calculated 2V:1H method agree reasonably well with the Boussinesq method for depths between B and 4B below the foundation.

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

Point Loading (Boussinesq 1883)



Stresses in an Elastic Medium Caused by Point Loading

Figure 6.11. Das FGE (2005).

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}\right\} = \frac{P}{z^2} I_1$$

Where:

$\Delta\sigma_z$ = Change in Vertical Stress

P = Point Load

$$I_1 = \frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}$$

***Based on homogeneous, elastic, isotropic infinitely large half-space**

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

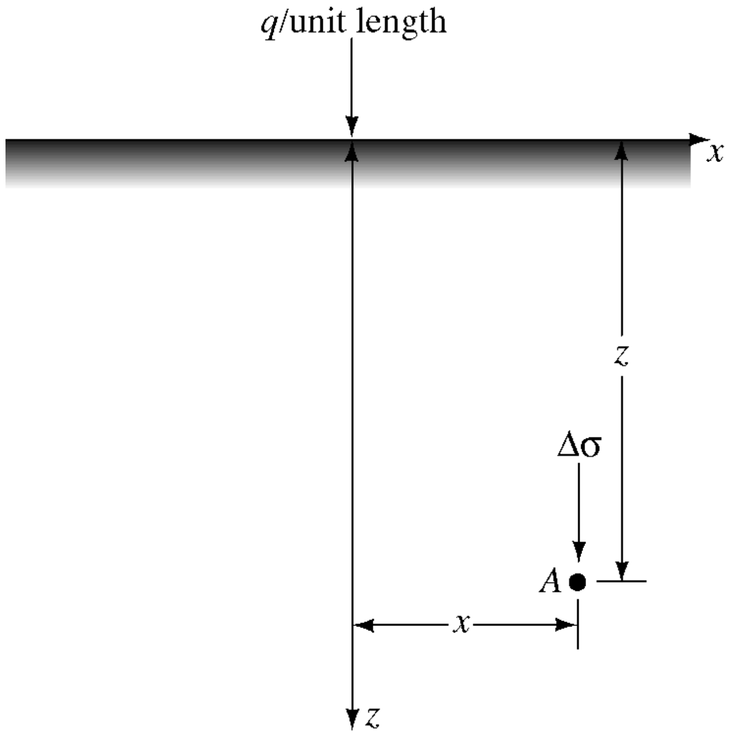
Point Loading (Boussinesq 1883)

Table 6.1 Variation of I_1 (Das, FGE 2006).

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

VERTICAL STRESS INCREASE ($\Delta\sigma_Z$) IN SOIL

LINE LOADING (BOUSSINESQ 1883)



Line Load over the Surface of a Semi-infinite Soil Mass
Figure 6.12. Das FGE (2005).

*Based on flexible line load of infinite length on a homogeneous, elastic, isotropic semi-infinite half-space

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2}$$

or

Dimensionless Form

$$\frac{\Delta\sigma}{(q/z)} = \frac{2}{\pi\left[\left(\frac{x}{z}\right)^2 + 1\right]^2}$$

Where:

- $\Delta\sigma$ = Change in Vertical Stress
- q = Load per Unit Length
- z = Depth
- x = Distance from Line Load

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

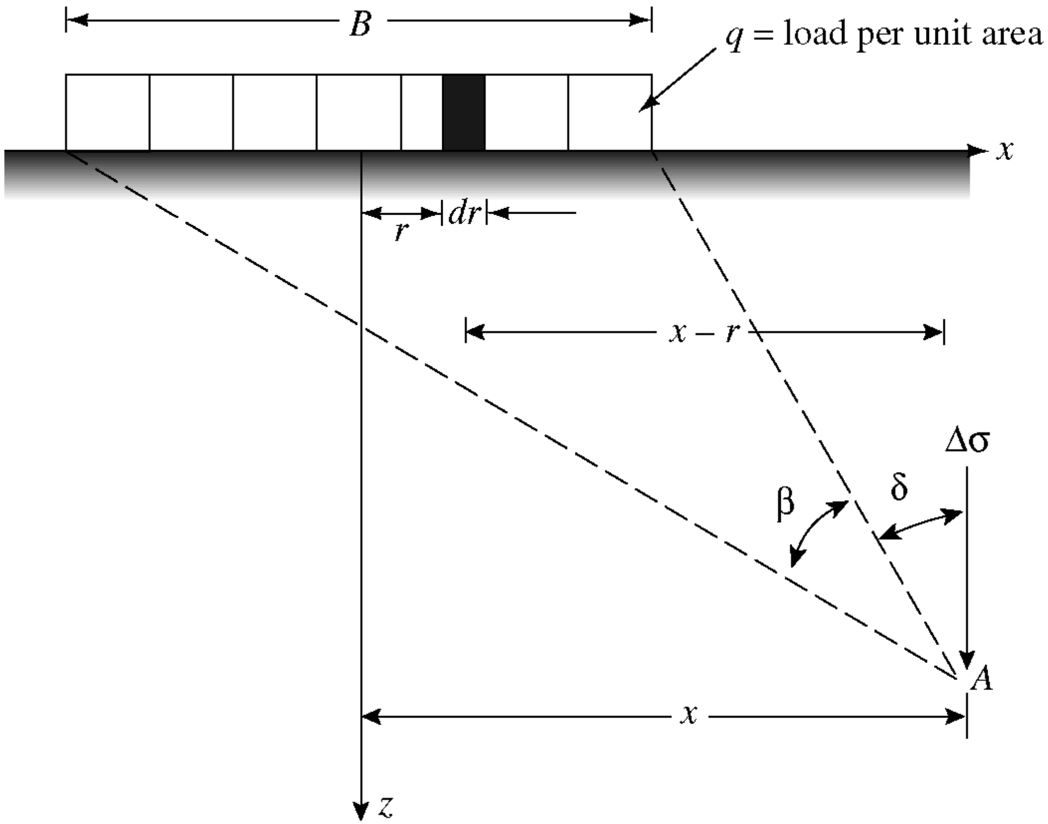
Line Loading (Boussinesq 1883)

Table 6.3 Variation of *$\Delta\sigma/(q/z)$* with *x/z* (Das, FGE 2006).

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

VERTICAL STRESS INCREASE ($\Delta\sigma_z$) IN SOIL

STRIP LOADING (BOUSSINESQ 1883)



Flexible Strip Load over the Surface of a Semi-infinite Soil Mass

Figure 6.13. Das FGE (2005).

$$\Delta\sigma = \frac{q}{\pi}\left[\beta + \sin\beta\cos(\beta + 2\delta)\right]$$

Where:

$\Delta\sigma$ = Change in Vertical Stress

q = Load per Unit Area

z = Depth

x = Distance from Line Load

Angles measured in counter-clockwise direction are taken as positive

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

Strip Loading (Boussinesq 1883)

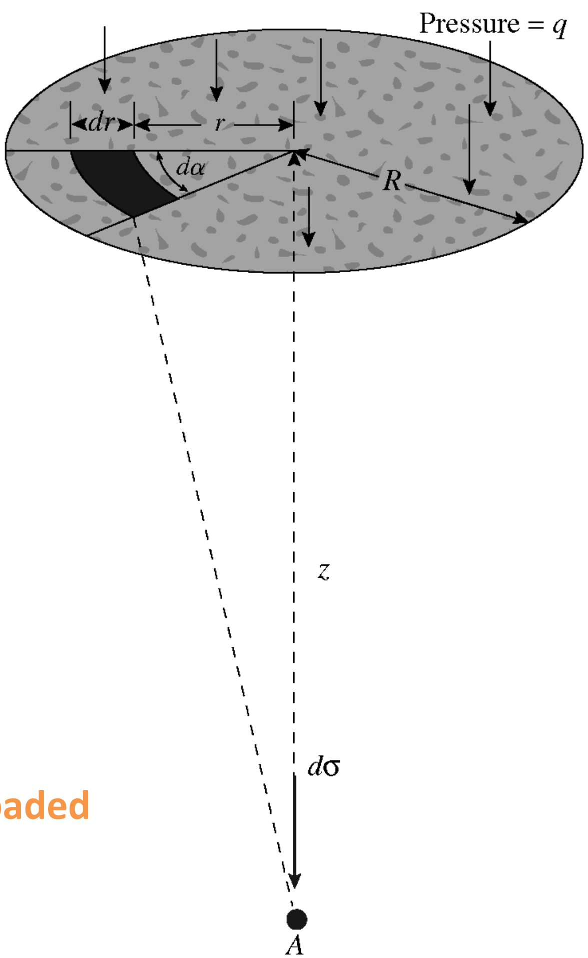
Table 6.4 Variation of $\Delta\sigma/q$ with $2z/B$ and $2x/B$ (Das, FGE 2006).

	2x/B				
2z/B	**0**	**0.5**	**1.0**	**1.5**	**2.0**
0	1.000	1.000	0.500	—	—
0.5	0.959	0.903	0.497	0.089	0.019
1.0	0.818	0.735	0.480	0.249	0.078
1.5	0.668	0.607	0.448	0.270	0.146
2.0	0.550	0.510	0.409	0.288	0.185
2.5	0.462	0.437	0.370	0.285	0.205
3.0	0.396	0.379	0.334	0.273	0.211
3.5	0.345	0.334	0.302	0.258	0.216
4.0	0.306	0.298	0.275	0.242	0.205
4.5	0.274	0.268	0.251	0.226	0.197
5.0	0.248	0.244	0.231	0.212	0.188

VERTICAL STRESS INCREASE ($\Delta\sigma_Z$) IN SOIL

CIRCULAR LOADING (BOUSSINESQ 1883)

$$Ds = q\left\{1 - \frac{1}{\left[(R/z)^2 + 1\right]^{3/2}}\right\}$$



Where:

- $\Delta\sigma$ = Change in Vertical Stress
- q = Load per Unit Area
- z = Depth
- R = Radius

Vertical Stress Below Center of Uniformly Loaded Flexible Circular Area

Figure 6.15. Das FGE (2005).

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

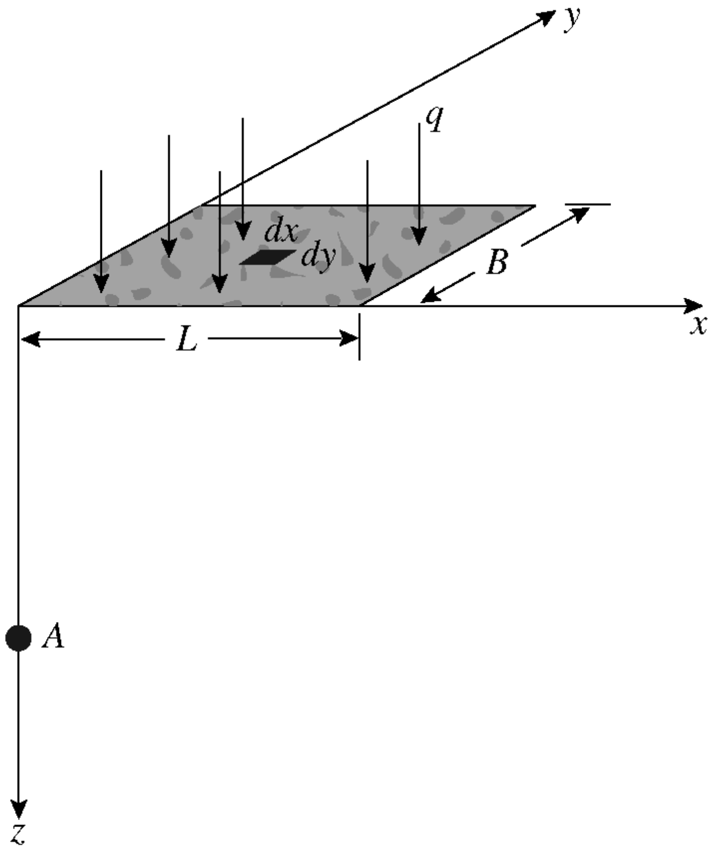
Circular Loading (Boussinesq 1883)

Table 6.5 Variation of $\Delta\sigma/q$ with z/R (Das, FGE 2006).

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

VERTICAL STRESS INCREASE ($\Delta\sigma_z$) IN SOIL

RECTANGULAR LOADING (BOUSSINESQ 1883)



Vertical Stress Below Corner of Uniformly Loaded Flexible Rectangular Area
Figure 6.16. Das FGE (2005).

$$\Delta\sigma = \int d\sigma = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dxdy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_2$$

Where:

- $\Delta\sigma$ = Change in Vertical Stress
- q = Load per Unit Area
- z = Depth

$$I_2 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$

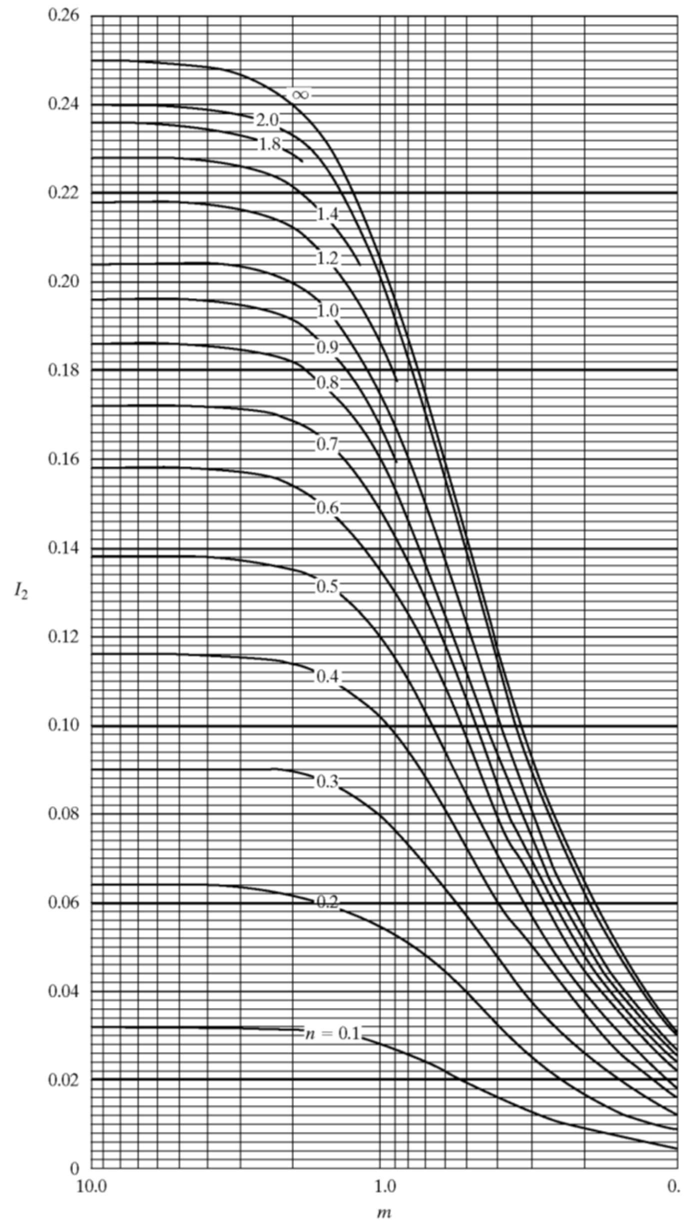
$$m = \frac{B}{z}; n = \frac{L}{z}$$

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

Rectangular Loading (Boussinesq 1883)

Variation of I_2 with *m* and *n*.

Figure 6.17. Das FGE (2005).



VERTICAL STRESS INCREASE ($\Delta\sigma_z$) IN SOIL

RECTANGULAR LOADING (WESTERGAARD)

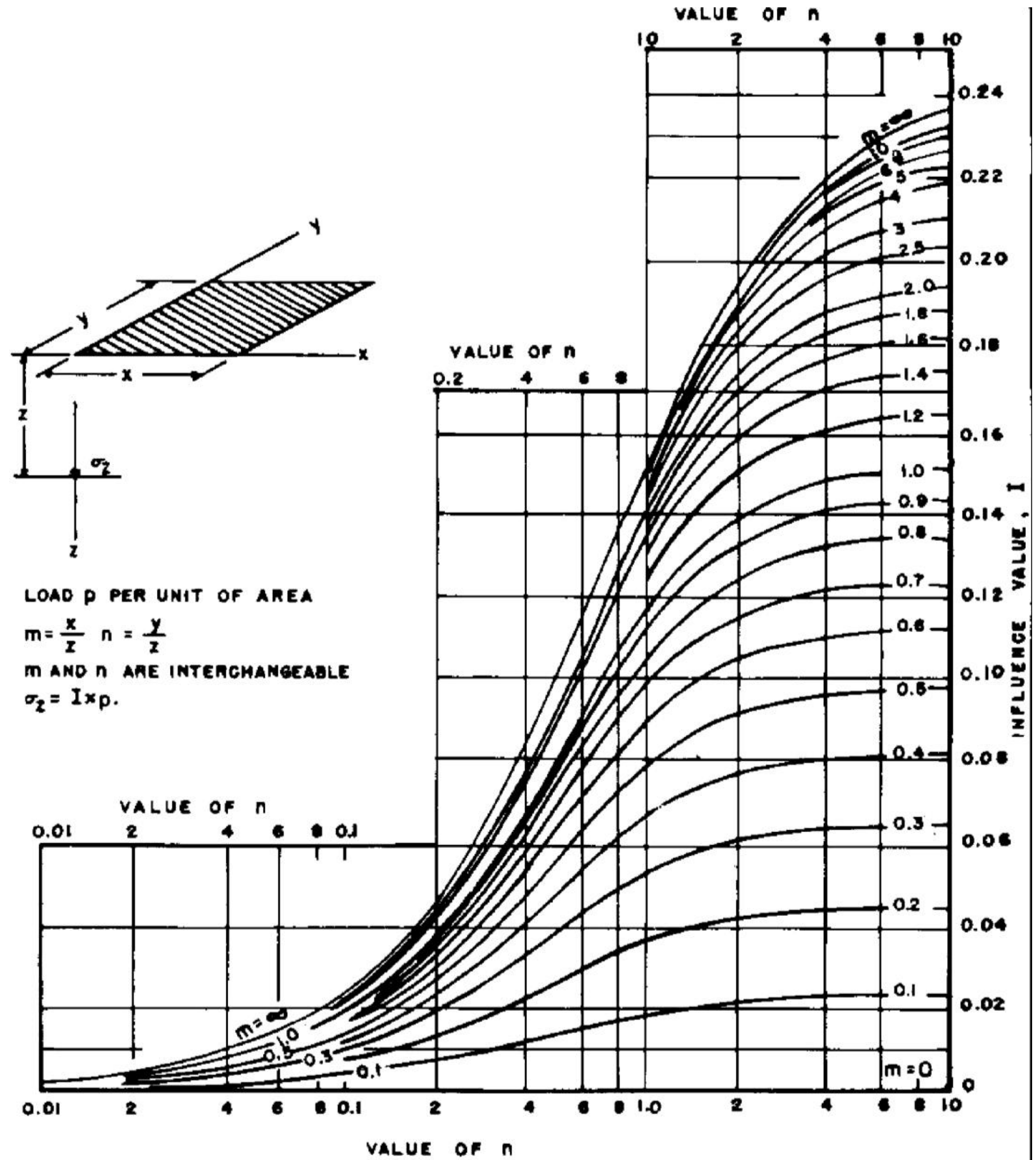


Figure 12. NAVFAC DM7.01.

VERTICAL STRESS INCREASE ($\Delta\sigma_z$) IN SOIL

RECTANGULAR LOADED AREA

Within a Rectangular Loaded Area:

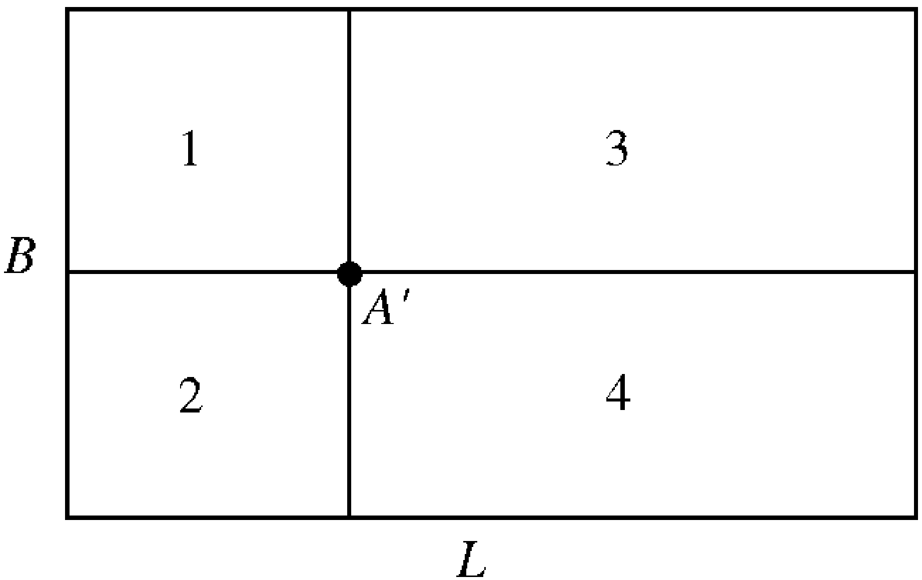


Figure 6.18. Das FGE (2005).

$$Ds = q\left[I_{2(1)} + I_{2(2)} + I_{2(3)} + I_{2(4)}\right]$$

Under Center of Footing:

$$Ds_c = qI_c$$

$$I_c = f(m_1, n_1)$$

$$m_1 = \frac{L}{B}; n_1 = \frac{z}{\frac{B}{2}}$$

Vertical Stress Increase ($\Delta\sigma_z$) in Soil

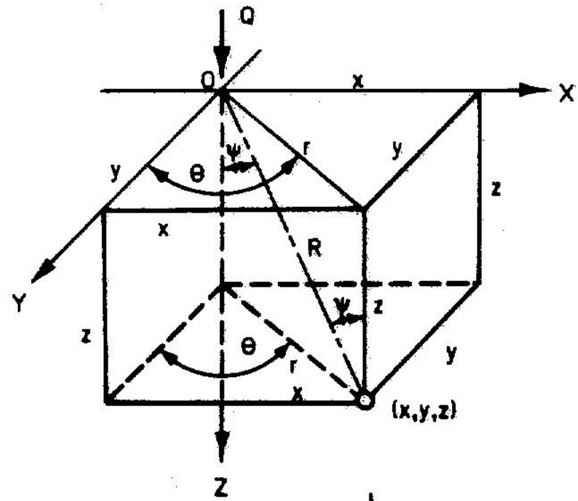
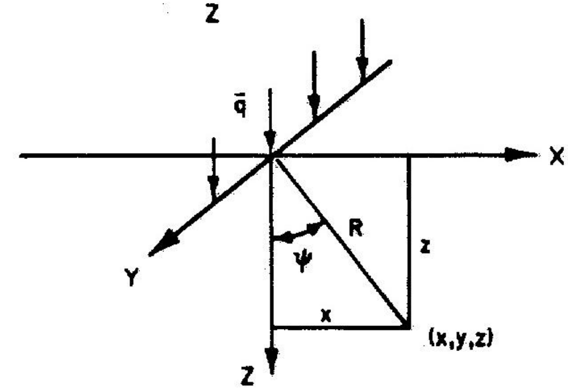
Center of Rectangular Loaded Area

Table 6.6 Variation of I_c with m_1 and n_1 (Das, FGE 2006).

	m_1									
n_1	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

BOUSSINESQ SOLUTIONS SUMMARY

(EM 1110-1-1904 TABLE C-1)

TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
POINT	$\frac{3Q}{2\pi R^2}\cos^3\psi$ Q = NORMAL LOAD, TONS $r^2 = x^2 + y^2$ $R^2 = r^2 + z^2$	
LINE	$\frac{2\bar{q}z^3}{\pi R^4}$ $\bar{q}$ = NORMAL LOAD, TONS/FT $R^2 = x^2 + z^2$	

BOUSSINESQ SOLUTIONS SUMMARY
(EM 1110-1-1904 TABLE C-1)

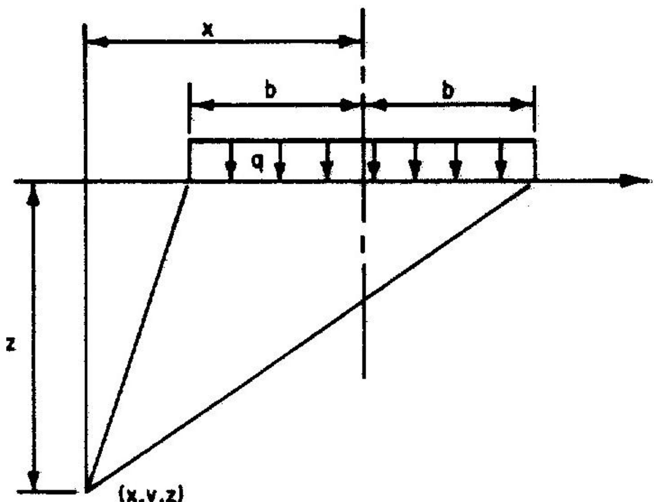
STRIP

$$\frac{q}{\pi}\left(\alpha + \mathrm{SIN}\,\alpha\,\mathrm{COS}\,(\alpha + 2\beta)\right)$$

q = CONTACT PRESSURE, TSF

$$\alpha = \mathrm{TAN}^{-1}\left(\frac{x+b}{z}\right) - \beta, \text{ RADIANS}$$

$$\beta = \mathrm{TAN}^{-1}\left(\frac{x-b}{z}\right), \text{ RADIANS}$$

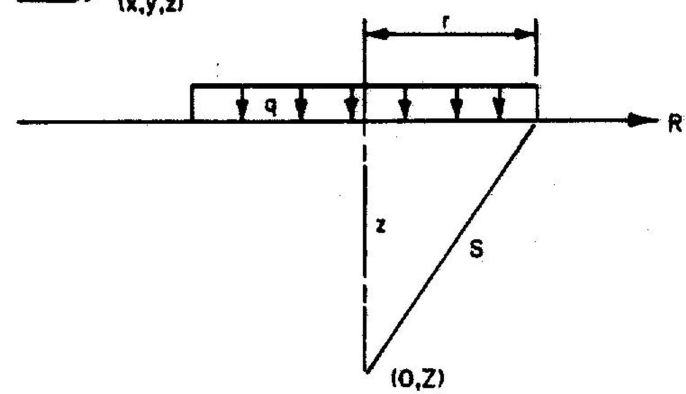


AREA (POINT UNDER CENTER CIRCULAR AREA)

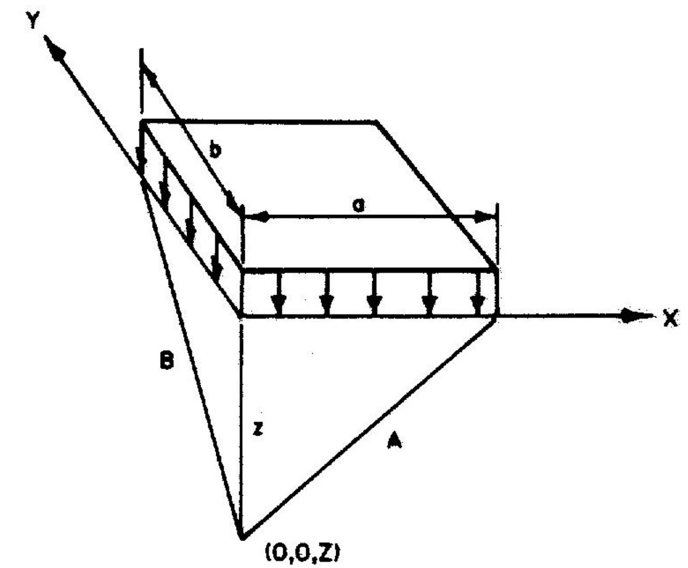
$$qr^2\,\frac{(S^2 + 2z^2)}{2S^4}$$

$$S^2 = r^2 + z^2$$

q = CONTACT PRESSURE, TSF

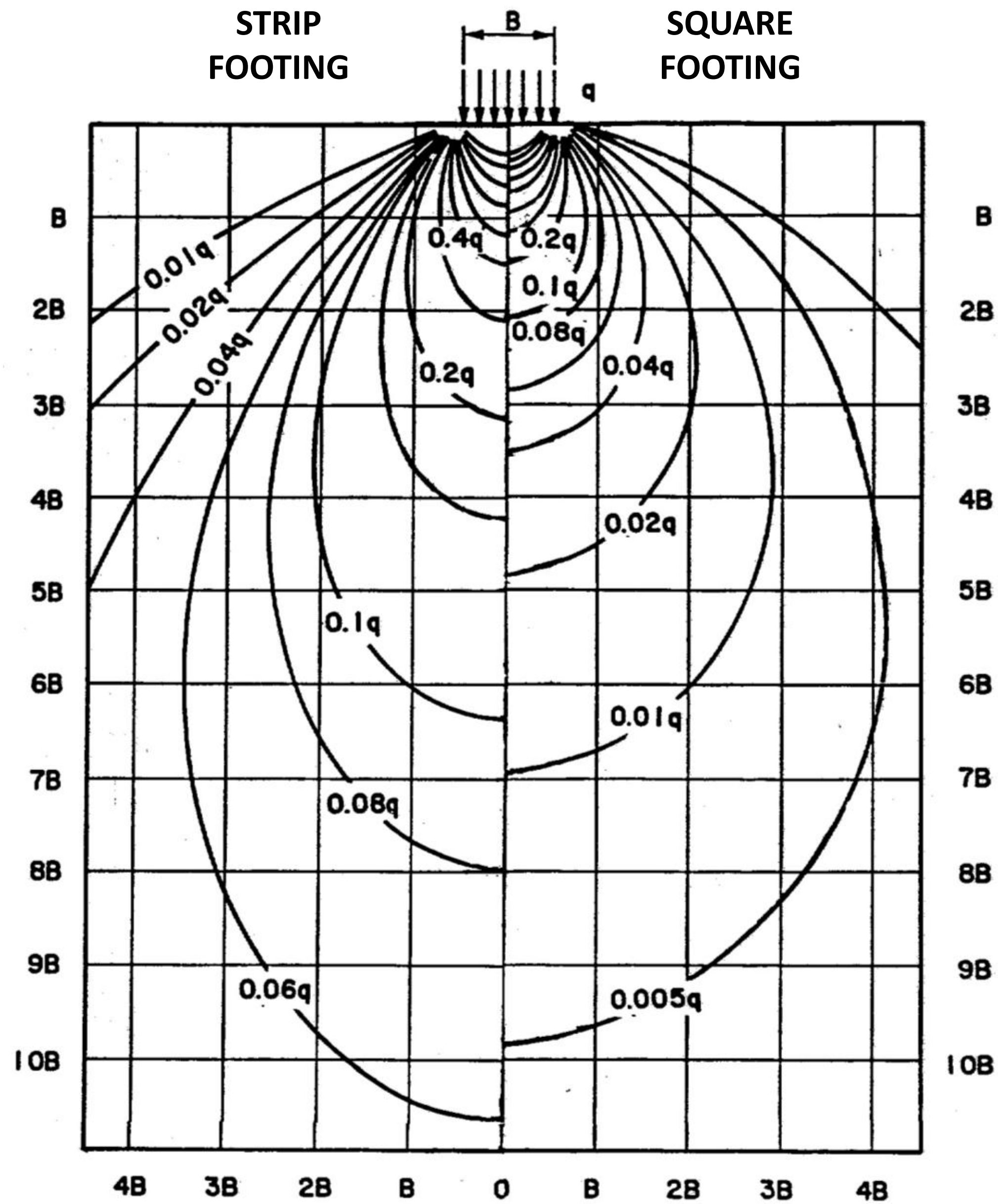


BOUSSINESQ SOLUTIONS SUMMARY
(EM 1110-1-1904 TABLE C-1)

TYPE OF LOAD NORMAL TO SURFACE	EQUATION FOR $\Delta\sigma_z$	COORDINATE SYSTEM
(POINT UNDER CORNER RECTANGULAR AREA)	$\frac{q}{2\pi}\left[\tan^{-1}\frac{a\,b}{z\,C} + \frac{a\,b\,z}{C}\left(\frac{1}{A^2}+\frac{1}{B^2}\right)\right]$ $A^2 = a^2 + z^2$ $B^2 = b^2 + z^2$ $C = (a^2 + b^2 + z^2)^{1/2}$ q = CONTACT PRESSURE, TSF	

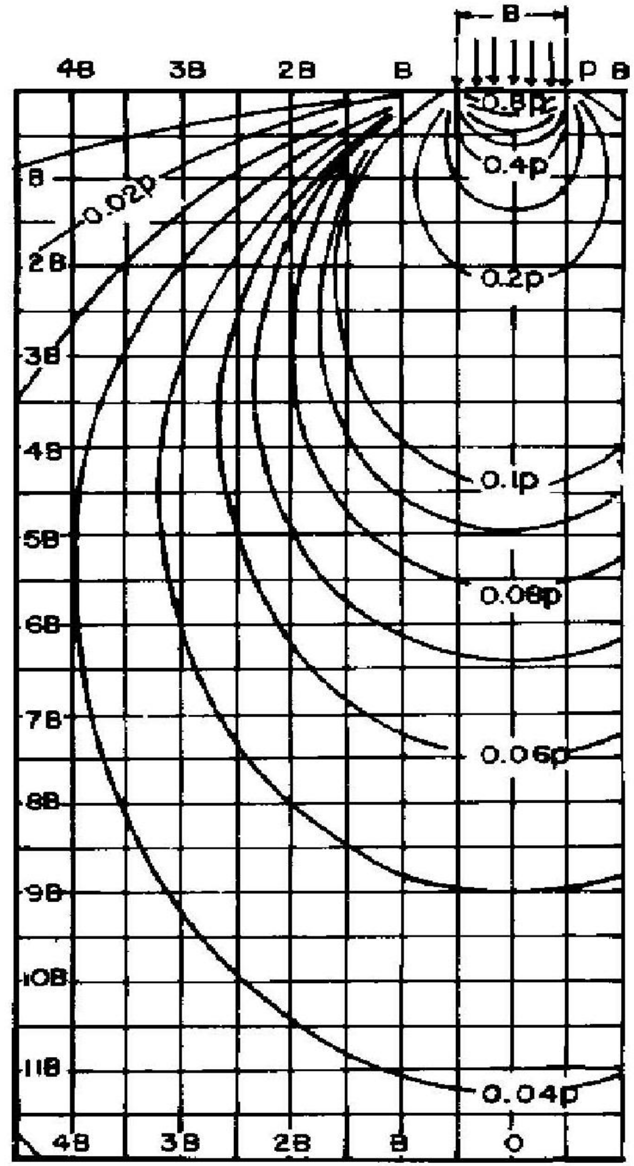
Boussinesq Graphical Solution

(EM 1110-1-1904 Figure 1-2)



Westergaard Graphical Solution

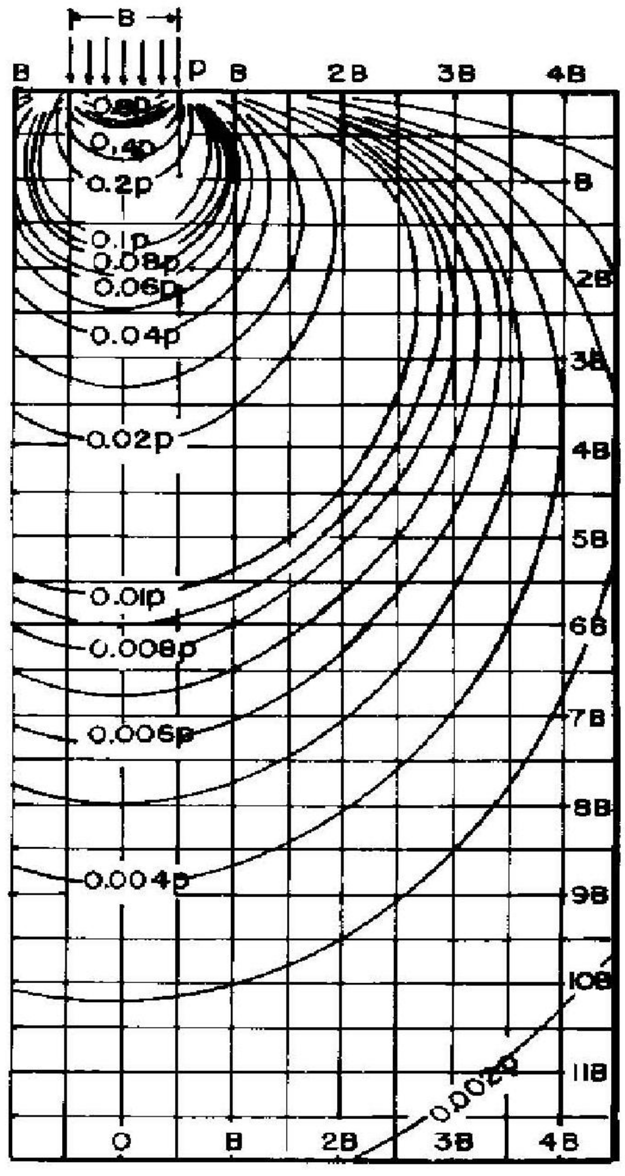
(NAVFAC DM7.01 Figure 11)



a. INFINITELY LONG FOOTING

WESTERGAARD GRAPHICAL SOLUTION

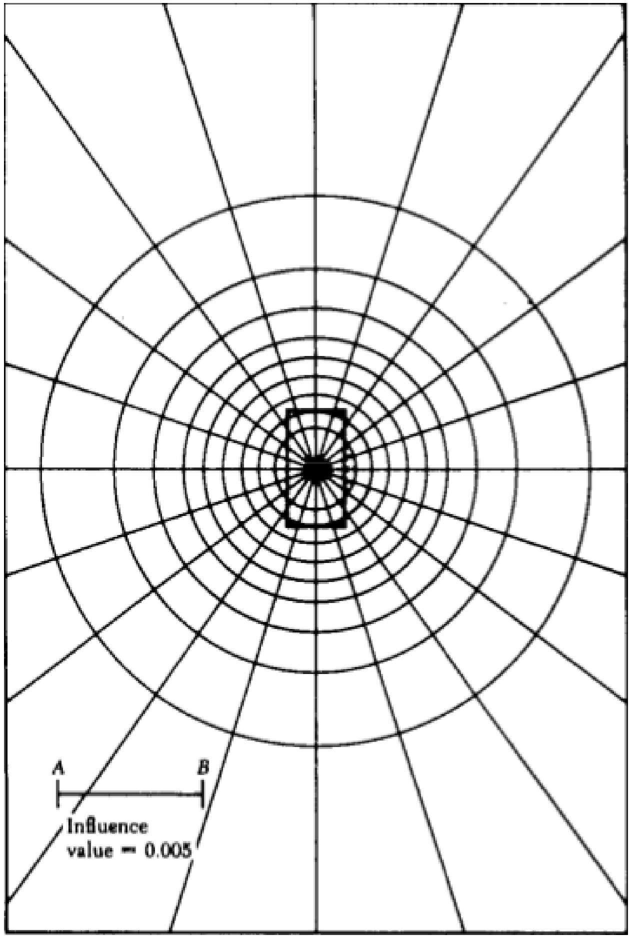
(NAVFAC DM7.01 FIGURE 11)



b. SQUARE FOOTING

NEWMARK INFLUENCE CHARTS

(BASED ON BOUSSINESQ SOLUTIONS)



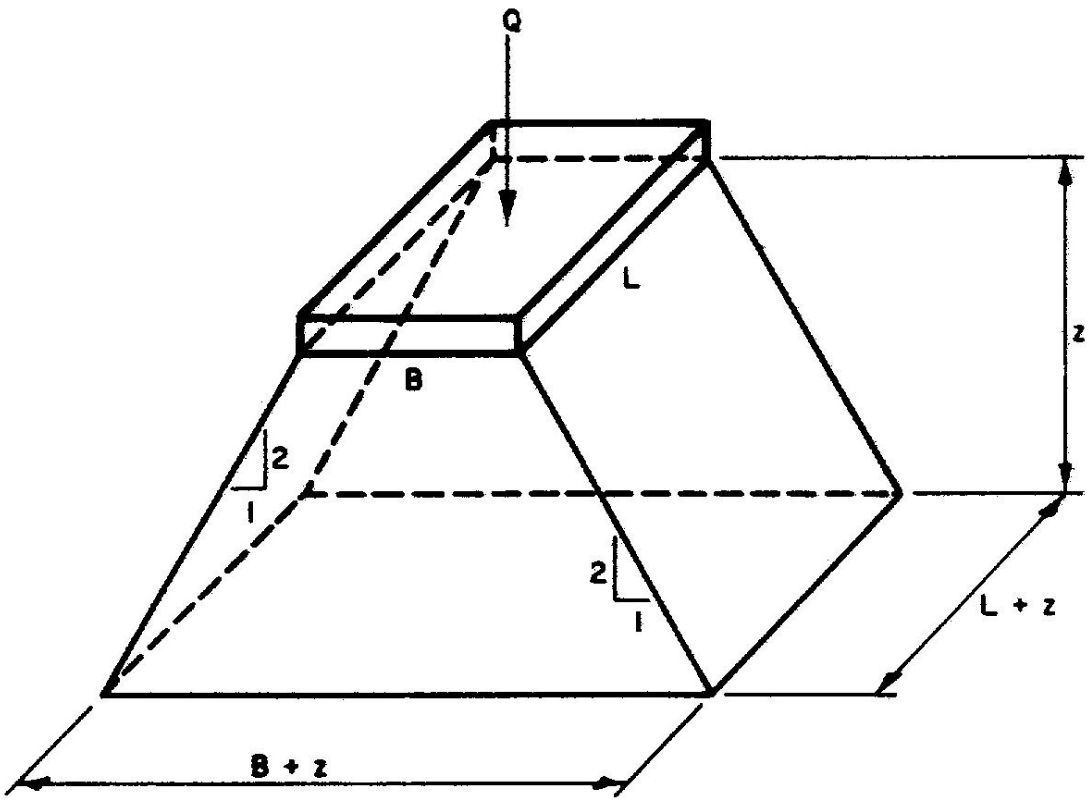
STEPS

1. Draw the footing shape to a scale using Length AB = Depth z.
2. The point under which we look for $\Delta\sigma_v'$, is placed at the center of the chart.
3. Count the units and partial units covered by the foundation (m).
4. $\Delta\sigma_v' = \Delta p = (q_o)(m)(I)$

I = Influence Factor

Vertical Stress Increases in Soil

Analysis Methods: 2V:1H Method



$$\Delta\sigma_z = \frac{Q}{(B+z)(L+z)}$$

Where:

$\Delta\sigma_z$ = Change in Total Vertical Stress

Q = Applied Foundation Load

B = Foundation Width

L = Foundation Length

Figure C-1. USACE EM1110-1-1904.